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**IDENTIFICATION OF LUMPED PARAMETER SYSTEMS**

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**Abstract** - In the interaction processes between space structures, instruments or equipment and the environment, as well as at contact surfaces between these structures, the heat transfer processes play an important role. Very often numerical simulation of these processes drives us to the necessity of using a heat transfer mathematical model with lumped parameters. In this case, complex heat transfer is considered for a system of bodies. In the case given the basic heat transfer equation is obtained from the analysis of a heat balance under the assumption that the considered system can be divided into a finite number of isothermic elements. One of the main difficulties here is how to determine coefficients of the mathematical model, which provide its adequacy to real processes. Direct measurement of most characteristics of heat transfer is usually impossible, and their theoretical estimates are often far from being true and often contradictory. That is why, a problem arises to determine the heat transfer characteristics of structures by means of calculations and experiments. The practical approval of the technique suggested has been carried out for the estimations of radiative characteristics of the spacecraft thermal control coatings. Of great interest is an experimental determination of the solar radiation integral absorptivity factor $A_s$ and integral semi-spherical emissivity $\varepsilon$ of space surface coating in the conditions of actual operation. Such studies were conducted on spacecraft of the "Cosmos", "Meteor", and "Meteor-Priroda" series.

## 1. INTRODUCTION

The problem of thermal control is amongst one of the most important problems arising in the development of space vehicles and supporting technology. As statistics show, with respect to failures in the operation of various systems and equipment of space vehicles, a great number of these malfunctions are connected with violations of admissible thermal conditions. On the other hand, when considering the interaction processes between space structures, instruments or equipment and the environment, as well as between the surfaces of these structures in contact, the radiative heat transfer phenomena plays an important role. To choose a rational thermal control system, it is necessary to analyze the radiative properties of coating materials. Very often a numerical simulation of such heat transfer processes drives us to a necessity of using a heat transfer mathematical model with lumped parameters. One of the main difficulties here is how to determine the coefficients of the mathematical model, providing its adequacy to real actions, [1]. Direct measurement of most heat transfer characteristics is usually impossible, and their theoretical estimates are often far from being true and often contradictory. That is why a problem arises to determine the heat transfer characteristics of structures by combining the calculations and the results of experiments. The algorithms suggested for specifying the unknown radiative characteristics of coating materials for spacecraft thermal control systems are based on the methodology of inverse heat transfer problems, which, at present, are widely used in the study of heat transfer processes. The computational methods for solving the boundary inverse heat conduction problems are now effectively used in experimental investigations of thermal processes occurred between the solids and the environment. The inverse problem methods allow the use of mathematically proven search algorithms for unknown heat transfer characteristics, estimated from the results of indirect measurements.

Many structures employed in various branches of engineering operate under conditions of strong thermal effects. The tendency is for extensive use of heat-loaded engineering objects and to high intensity of heat conditions. At the same time, it has been necessary to increase the reliability and service life of goods, while reducing the specific consumption of materials. The non-stationary state and non-linearity (considerable, at times) of the heat transfer phenomena can be referred to special features of thermal conditions of modern thermo-loaded structures. These factors considerably reduce the possibility of using many traditional theoretical and experimental methods. So it became urgent to develop new approaches to thermal engineering studies. Amongst such approaches are methods based on the solution of inverse problems, in which it is required, through measurements of the system or process state, to specify one or several characteristics which cause this state (in other words, to find not causal-sequential, as in direct problems, but rather sequential-causal quantitative relations). The advantage of these methods is that they help to carry out experimental studies under conditions very similar to full-scale tests or in the operation of the considered systems, in particular in flight tests. In addition, new information gained from these studies makes it possible to speed up the experimental methods and reduce the cost. This is very important for structures used in the space industry, and we observe this situation in the field of practical applications where the first formulations and methods of solving the inverse heat transfer problems have appeared. Experimental-and-computational methods based on solving inverse heat coefficient transfer problems form an intensively developing direction in the field of studies of heat transfer processes, [2,3,6,7,9,10].

The purpose of this paper is to introduce a new method in the research of radiative properties of materials with further applications in the design of thermal devices to condition a thermal environment for the payload boxes in satellites. The effectiveness of operation of such devices depends on the properties of materials used. Taking into account a relative significance of thermal radiation in heat interactivity between flight vehicles and space environment, we need to estimate the radiative properties of materials for thermal coatings with the required accuracy.

Formulations of the transient heat transfer problem usually assume variations in the temperature both in time and in position. However there are many engineering applications in which the variation of temperature within the medium can be neglected and the temperature is then considered as a function of time only. Such a formulation of the problem, called a lumped system formulation, provides greater simplification in the analysis of transient heat transfer although the range of applicability is rather restricted. Here we illustrate a concept of the lumped formulation approach, examining its range of validity.

In this situation, the complex heat transfer in a spacecraft or in any other engineering system is considered for a system of elements, which exchange thermal energy with the environment and between themselves. The basic heat transfer equation is obtained from the analysis of heat balance under the assumption that the engineering system can be divided into a finite number $L$ of isothermic elements:

$$C_l(T,\tau)\frac{\partial T_l}{\partial \tau} = \sum_{j=1}^{L} Q_{lj}(\tau)(T_j - T_l) + \sum_{j=1}^{L} R_{lj}(\tau) F_l \sigma \left(T_j^4 - T_l^4\right) + E_l(T,\tau) + S_l(\tau),$$
$$\tau \in (\tau_{\min}, \tau_{\max},], \quad l = \overline{1, L}; \tag{1}$$

$$T_l(\tau_{\min}) = T_{l0}, \quad l = 1,2,\ldots,L, \tag{2}$$

where $C_l$ is the total heat capacity of the $l$-th element, $Q_{lj}$ is the heat exchange coefficients between elements $l$ and $j$, $R_{lj}$ is the radiative-angular coefficients, $\sigma$ is the Stephan-Boltzmann coefficient, $E_l$ is the heat quantity supplied to $l$-th element from the environment, $S_l$ is the heat quantity released in $l$-th element, $l$ and $j$ the number of elements interacting with the environment via convection, conduction and radiation, respectively, and $F_l$ is the surface area of the element. The coefficients $Q_{lj}$ are calculated through different relations depending on the kind of heat exchange between elements $l$ and $j$, in particular of thermal conduction or convection.

In the general case, the heat transfer process described by Eqns. (1-2) is determined by the parameters of the boundary conditions and heat balance equations, by heat transfer of the conductive, convective and radiative kind, by thermal properties and heat sources, by geometry and the relative position of elements, as well as by the system's initial thermal state. Usually the corresponding parameters are referred to as the one so called causal characteristics of the heat transfer process under consideration. If we are to calculate the thermal state (temperature-time dependence) of a system by given causal characteristics, such a calculation will be an objective of the direct heat transfer problem in the system. In the case that some of the causal characteristics are not known and it is required to define them using available information about the thermal state of the system (true for simulation and admissible for design). Then we solve an inverse heat transfer problem. The model as in Eqns. (1-2) is often used in the description and optimization of heat transfer processes in various engineering systems (electrical engineering and radioelectronic appliances, flight vehicle modules, various heat exchangers, etc.). Hence, the inverse problems for Eqns. (1-2) can be called inverse problems of heat transfer in the engineering systems [1].

One should note that a simultaneous determination of all parameters $C_l$, $S_l$, $R_{lj}$, $Q_{lj}$, $E_l$ is possible only with an accuracy up to a constant multiplier. The uniqueness of solutions is extremely important in our studies involving the inverse problem solutions since the uniqueness theorems ensure determinacy of the solution and informativeness of experiments [1]. Moreover, in view of Tikhonov's classical theorem the uniqueness theorems ensure the computation stability if the solution of inverse problems is sought on the compact. The uniqueness of the solutions of the corresponding problem was analyzed by Nenarokomov [8].

**2. NUMERICAL ALGORITHM**

Let us suppose that in a real situation there are some unknown characteristics $u_i,\ i=1,2,...,N$ among elements of vectors $\{C_l\}_1^L, \{E_l\}_1^L, \{S_l\}_1^L, \{T_{l0}\}_1^L$ and matrixes $\{Q_{jl}\}, \{R_{jl}\},\ j,l=1,2,...,L.$

In addition, the results of temperature measurements in the separate elements of the system are available

$$T_l^{\exp}(\tau_m) = f_{lm},\ m=\overline{1,N_l},\ l=\overline{1,L}. \tag{3}$$

One of the most promising directions in solving the inverse heat transfer problems is to reduce them to extremal formulations and apply numerical methods of the optimization theory. In the exact extremal statement, the definition of functions $u_i,\ i=1,2,...,N$ corresponds to a minimization of the residual functional characterizing the deviation of temperature $T_l\ (\tau_m)$ calculated for certain estimates of $u_i,\ i=1,2,...,N$ from known temperature $f_{lm}$ in the metric of space of the input data (the mean-square deviation of experimental $f_{lm}$ and theoretical $T_l\ (\tau_m)$ temperatures can be used as a functional):

$$u = \arg\min_{u\in L_2} J(u) \tag{4}$$

where

$$J(u) = \sum_{l=1}^{L}\sum_{m=1}^{M_l}\left(T_l(\tau_m) - f_{lm}\right)^2.$$

A theoretical temperature $T$ is calculated using the mathematical model Eqns. (1-2). Such an inverse problem is known to belong to a class of ill-posed problems in their classical sense. More often the ill-posedness is stipulated by the instability of the problem solving with respect to small perturbations of the input data. Despite this feature, it is possible to solve an inverse problem using one or other regularization method. Among them an iterative regularization method is one of the most universal and efficient [1]. The method is based on gradient iterative algorithms, in which, and this is very important, the last iteration number is chosen according to the residual principle [1]. To solve the inverse problem (4), the following iterative method of unconstrained minimization can be used:

$$u^s = u^{s-1} + \gamma^s g^s,\ \ s=1,...,s^*,$$

$$g^s = -J_u^{'} + \beta^s g^{s-1},\ \ g^0 = 0,$$

$$\beta^s = \left\langle J_u^{'}\left(u^s\right) - J_u^{'}\left(u^{s-1}\right)\right\rangle_{L_2} \Big/ \left\| J_u^{'}\left(u^s\right)\right\|_{L_2},\ \ \beta^0 = 0\ . \tag{5}$$

The last iteration number $s^*$ is chosen according to the iterative residual principle. As the input data measuring and registering may be performed using inaccurate technical equipment, so the data obtained may be approximate, and an inverse problem solution corresponding to the input data, already measured, may differ greatly from a true one and carry a pronounced oscillating nature. Also a discretization process inevitably leads to errors conditioned by the approximation of continuous functions through piecewise-polynomial dependences, as well as by approximation of differential and integral operators through difference and summation operators. Moreover, errors of rounding-off occur when performing arithmetic operations. All these errors give instability of results in approaching the minimum of the residual functional. It is possible to suppose that methods enabling effective initiation of the iterative process from distant approximations $u_i,\ i=1,2,...,N$ and the sharp slow down in approaching the functional minimum would appear useful when solving inverse heat transfer problems. Such a method of instability damping when specifying the approximate solution for an ill-posed problem is based on the "viscous" properties of numerical optimization algorithms. It is necessary to keep in mind that as the number of iterations increases an inverse problem solution can worsen, gradually losing its smooth character. Any waviness appearing in $u_i,\ i=1,2,...,N$, will gain in strength as fast as the increasing fluctuating errors burden the measured temperature data and the greater would be the sensitivity of $u_i,\ i=1,2,...,N$ to the temperature measurements. Here we suggest stopping the iterative process at a certain iteration $s^*$, admitting no oscillation in the solution.

The main question in such an approach is how to select a stopping criterion. With this in mind, let us introduce a complementary condition specifying an admissible degree of proximity of the approximation sought for the "exact" functions $u_i,\ i = 1,2,...,N$, which corresponds to the disturbed measured data. A condition as such can be a restriction to a residual level given as an aggregate error, including an error in temperature measurements and an approximation error (discretization) in the direct heat transfer problem. A theoretical analysis of different iterative methods has permitted the establishment of the following results. First of all, the gradient methods, such as the steepest descent, the minimal errors, the simple iteration and the conjugate gradients generate the regularizing families of operators. That is, it becomes possible to choose a stable approximation to the unknown solution from the corresponding iterative sequence. Secondary, if iterations are stopped on the basis of residual criterion, these methods are the regularization algorithms, that is, they give stable approximate solutions whose accuracy increases steadily as the errors of the input data are reduced. These rigorous mathematical results were obtained for a linear case [1]. Thus, let us bound the iterative sequence Eqn. (5) according to the condition

$$J\left(u^{s^*}\right) \le \delta_f^2 \tag{6}$$

where $\delta_f^2$ is the mean-square temperature-measurement error, namely

$$\delta_f^2 = \sum_{l=1}^{L}\sum_{m=1}^{M_l}\sigma_{lm}^2$$

The descent parameter $\gamma^s$ is determined from a condition

$$\gamma^s = \arg\min_{\gamma \in R^+}\left(J\left(u^s + \gamma\, g^s\right)\right). \tag{7}$$

Accordingly in the approach suggested in [1] the unknown coefficients in Eqn. (1) can be approximated by some systems of basic functions (in particular B-splines), for example

$$C_l(T) = \sum_{k=1}^{N_{C_l}} c_k^l \varphi_k^{C_l}(T), \qquad E_l(T,\tau) = \sum_{k=1}^{N_{E_l}} e_k^l \varphi_k^{E_l}(T,\tau), \qquad S_l(\tau) = \sum_{k=1}^{N_{S_l}} s_k^l \varphi_k^{S_l}(\tau)$$

$$Q_{j,l}(\tau) = \sum_{k=1}^{N_{Q_{j,l}}} q_k^{j,l} \varphi_k^{Q_{j,l}}(\tau), \qquad R_{j,l}(\tau) = \sum_{k=1}^{N_{R_{j,l}}} r_k^{j,l} \varphi_k^{R_{j,l}}(\tau) \tag{8}$$

The gradient of the minimized functional is computed using the solution of a boundary-value problem for an adjoint variable:

$$J'_{c_k^l} = -\sum_{m=1}^{M_l}\left(\varsigma_{ml}(\tau_m)\varphi_k^{C_l}(T)\frac{d\mathrm{T}_l}{d\tau}\right), \quad J'_{e_k^l} = -\sum_{m=1}^{M_l}\left(\varsigma_{ml}(\tau_m)\varphi_k^{E_l}(T)\right), \quad J'_{s_k^l} = -\sum_{m=1}^{M_l}\left(\varsigma_{ml}(\tau_m)\varphi_k^{S_l}(\tau)\right)$$

$$J'_{q_k^{j,l}} = -\sum_{m=1}^{M_l}\left(\varsigma_{ml}(\tau_m)(T_j - T_l)\,\varphi_k^{Qj,l}(\tau) - \varsigma_{mj}(\tau_m)(T_l - T_j)\,\varphi_k^{Qj,l}(\tau)\right)$$

$$J'_{r_k^{j,l}} = -\sum_{m=1}^{M_l}\left(\varsigma_{ml}(\tau_m)(T_j - T_l)\,\varphi_k^{Rj,l}(\tau) - \varsigma_{mj}(\tau_m)(T_l - T_j)\,\varphi_k^{Rj,l}(\tau)\right) \tag{9}$$

where $\varsigma_{ml}$ is the solution of the following adjoint problem:

$$C_l(T,\tau)\frac{d\varsigma_{ml}}{d\tau} = \sum_{j=1}^{L} Q_{lj}\left(\varsigma_{mj} - \varsigma_{ml}\right) + \sum_{j=1}^{L} R_{lj} F_l \sigma 4\left(\varsigma_{mj}^3 - \varsigma_{ml}^3\right)$$
$$+\frac{dE_l}{d\tau}\varsigma_{ml},\ \tau \in \left(\tau_{m-1}, \tau_m\right),\ m = \overline{1, M_l},\ l = \overline{1, L}, \qquad (10)$$

$$\varsigma_{M_l+1,l}\left(\tau_{\max}\right) = 0,\ l = \overline{1, L}. \qquad (11)$$

**3. APPLICATIONS**

The creation of reliable thermal protected covering for optical research and control equipment in satellites "Meteor" and "Vega" was a very important scientific and technical problem which demanded providing a complex experimental-computational research of the corresponding heat transfer processes. A significant part of this research was based on the results of flight testing, in particular, on the analysis of stability of radiative characteristics of materials. The necessity in such research is explained by various events of anomalies, appearing during flight tests of satellites (usually resulting in essential discrepancies between predicted and measured temperatures). These phenomena can be explained by the degradation of the surface of materials: after the increasing of its absorptivity under interaction with the environment. To prove this theory a special experimental approach to the identification of absorptivity $A_s$ and emissivity $\varepsilon$ in the real mode of operation was suggested. Based on these results adequate mathematical models for the long-time prediction of radiative properties of materials can be created.

There are no methods reliable enough for direct measurement of coefficients $A_s$ and $\varepsilon$, and so the suggested method is based on the inverse problem theory. Special specimens with different thermal coatings were made and installed on the outline zones of the external surface of satellites, neglecting the influence of radiative and conductive heat transfer with the elements of a structure. As an example an experimental specimen of satellites "Meteor-2" (Figure 1) is presented in Figure 2. This specimen consists of an aluminum (1) alloy slab 120x80x2mm. A thin film of the testing material is installed on the external surface of the slab and a film thermal sensor (2) with corresponding plugs is installed on the internal surface. A slab (1) via insulated cylinders (4), disks (3) and screen-vacuum thermal insulation (5) is fixed to a structure of the satellite (6,7,8).

The experiments were executed during flight testing of the satellites [4]. The altitude of orbits was about 900 km. In this case the temperature of specimens (taking into account the insulation from a structure of the satellite) is determined by direct solar radiation $q_s$, by solar radiation reflected from the Earth $q_R$ and by the Earth's irradiation $q_\varepsilon$.

If the Biot number is Bi << 0.1, the thermal resistance of the analyzed system can be neglected. Also if the relation for two layers of specimen $d_1\rho_1 C_1 / d_2\rho_2 C_2 > 10$ is realized, it is possible to consider only that heat which has been accumulated by the metal slab, neglecting heat capacity of the analyzed material. Then, a heat transfer mathematical model in the testing module under consideration can be presented as follows:

$$d_2\rho_2 C_2\, dT_w / d\tau = +A_s\left(q_s(\tau) + q_R(\tau)\right) + \varepsilon\, q_\varepsilon(\tau) - \varepsilon\,\sigma\, T_w^4,$$
$$T\left(\tau_{\min}\right) = T_{0.} \qquad (10)$$

The values of radiative heat fluxes are determined using the usual approach [5]. Let a satellite move around the Earth (radius $R$) at altitude $H$, see Figure 3. At a point of the surface of the orbiting body with fixed orientation the positive direction of the $Z$ axis is the vector from the center of the Earth to the mentioned point and is perpendicular to the outward surface normal, the positive direction of $Y$ axis being the velocity vector of the satellite. The angle $G$ is the angle between the normal from the orbiting surface and the direction from the specimen to the sun. The position of the satellite in orbit is defined by the angle $\omega\tau$, where $\omega$ is its angular velocity. Also we will use $\gamma$ - angle between vectors from the center of the Earth to the specimen surface and to the sun, $\varsigma$ - the angle between the outward surface normal vector and radius-vector of the Earth. Therefore,

$$\gamma = A\cos\left(\sin G\ \cos\omega\tau\right)$$

and

$$\varsigma = \pi / 2.$$

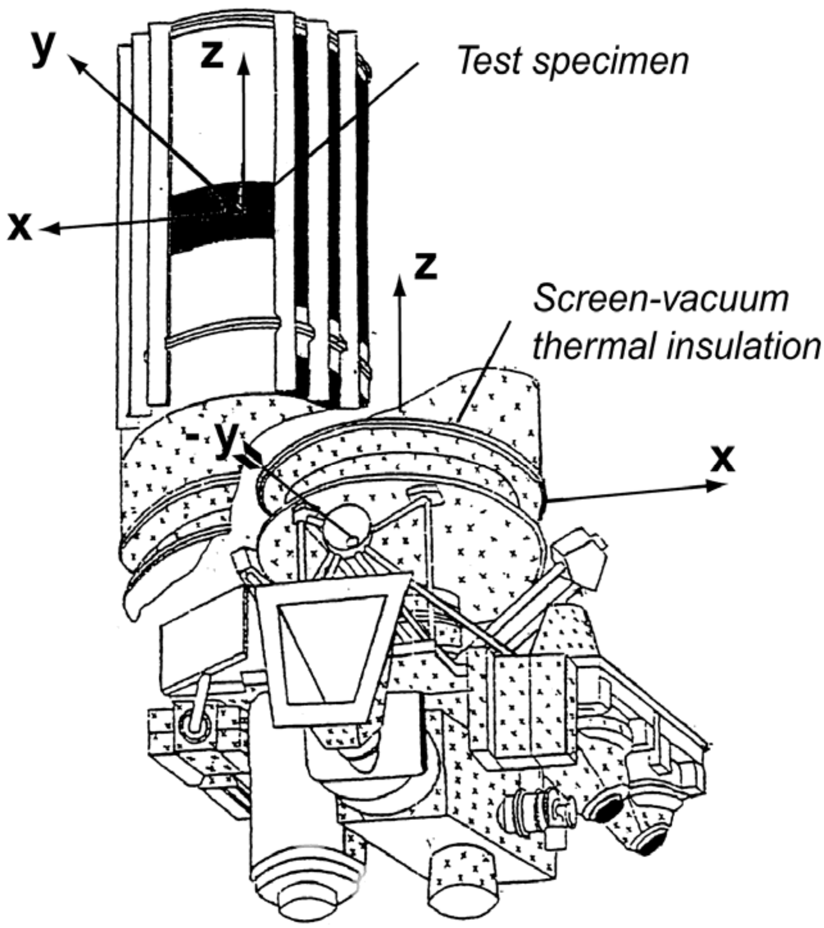


Figure 1. Satellite "Meteor-2" with testing specimens of materials

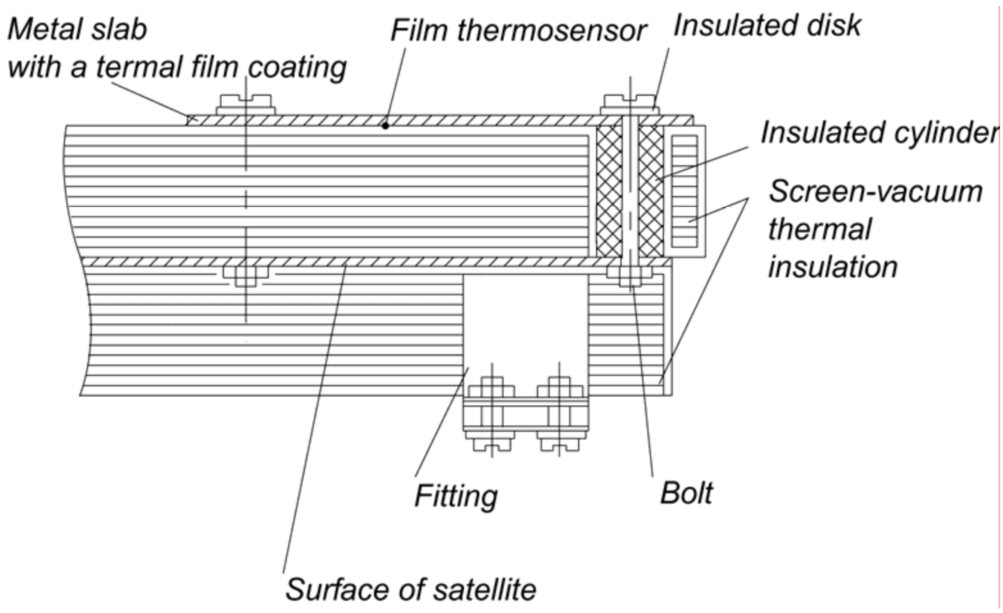


Figure 2. Experimental module: 1 - metal slab with a thermal coating, 2 – film thermosensor, 3 - insulated disk, 4 - insulated cylinder, 5 - screen-vacuum thermal insulation, 6 - bolt, 7 - fitting, 8 - surface of satellite.

If a satellite is in orbit opposite to the dark side of the Earth, $q_s$ and $q_R$ are equal to 0. The condition

$$\gamma > (\pi - \theta)$$

where $\theta = A\sin(R/(R+H))$ ensure that only the radiation from the sunny part of the Earth is taken into account. Hence

$$q_s = s_0\left(1-\sin^2 G\cos^2 \omega\tau\right)^{1/2} \qquad (11)$$

where $s_0$ is the solar radiation,

$$q_\varepsilon = (1-4\alpha)/4s_0/(2\pi)(2A\sin(R/(R+H)) - \sin(2A\sin(R/(R+H)))) \qquad (12)$$

where $\alpha$ is the albedo of the Earth and

$$q_R = \alpha\, s_0 \varphi_r \qquad (13)$$

where $\varphi_r$ is a complex angle coefficient.

If $\gamma < 60^\circ$ a diffuse model of reflection is used, where as if $\gamma > 60^\circ$ a specular model of reflection is used.

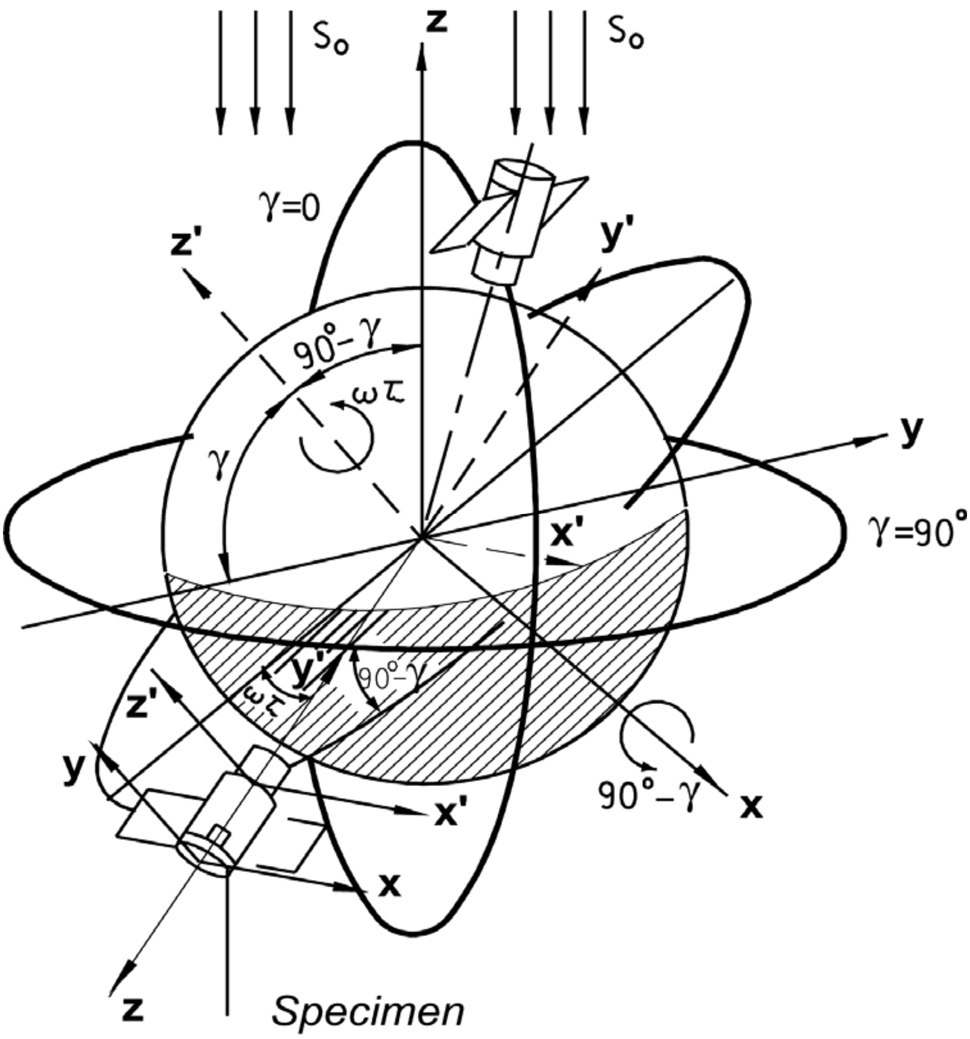


Figure 3. Parameters of the orbit.

Some experimental data obtained from a "Meteor-2" mission is considered in the present paper. When the experiments were carried out the temperature was measured at the times $\tau_m,\ m=\overline{1,M}$ . A representative set of measurements in one rotation of the satellite round the orbit is shown in Figure 4. A set of $f_1,..., f_M$ presents the 43 measurements with a time step of 143 seconds and in a time interval of 6152 seconds. The measurement errors do not exceed $5.7\,K$ , $\sigma_f < 5.7\,K$ .

The following initial data was used: $s_0 = 1396 W/m^2$, $\alpha = 0{,}39$, $H = 90\,km$, $G = 84^\circ$, $d_1 = 0{,}0025 m$, $d_2 = 0{,}00002\,m$, $\rho_2 = 2700\,kg/m$.

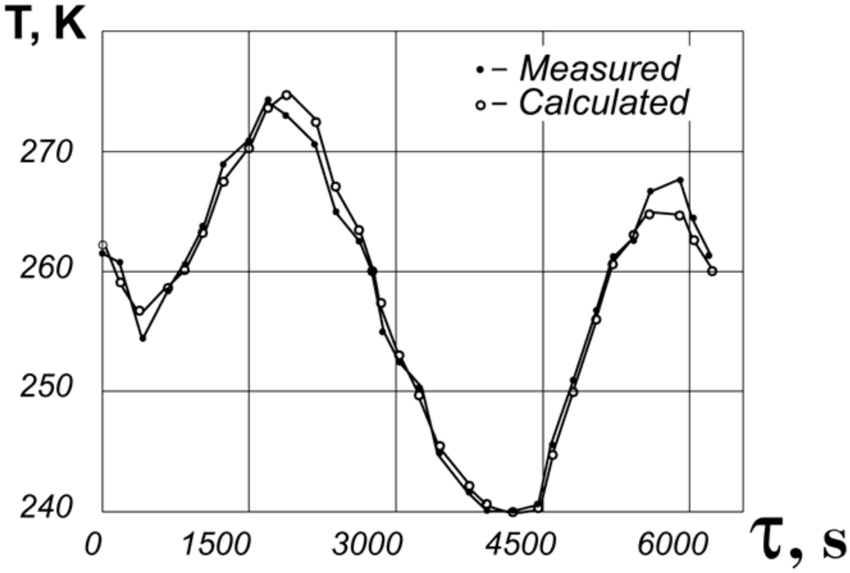


Figure 4. Temperature as a function of time: 1 - experimental, 2 - calculated.}

The uniqueness of the results obtained is confirmed from the different initial values of the desired parameters in the iterative procedure, see Figure 5. The results of solving the corresponding direct problem are presented in Figure 4, the temperature values were calculated using the obtained estimates of $A_s = 0.183$ and $\varepsilon = 0.771$ .

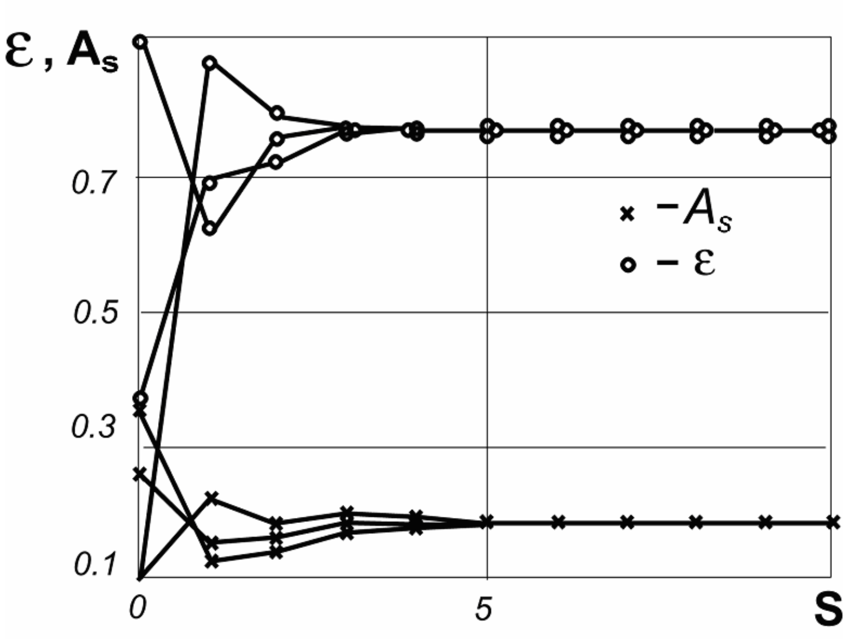


Figure 5. Iterative solution of inverse problem: 1 -absorptivity estimates, 2 - emissivity estimates.

## 4. CONCLUSIONS

The paper aims to describe the algorithm developed to process the data of flight-testing thermal experiments and hence to find the radiative properties of the thermal control materials of the surface. The algorithm is suggested for estimation of these unknown properties of the surface as a solution of the nonlinear inverse heat conduction problems in extreme formulation as well as for optimal design of corresponding experiments. The illustrative examples are presented to make a judgment on the convergence for real technical problems.

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